Let $(Y,B)$ be a "log Fano pair" such that the "positivity" to take under considerations to define "D-log K-stability". The "Donaldson-Futaki" invariant is defined as $DF(\omega;D)$ and it vanishes if and only if $\omega\in\mathcal{M}(X)$.

We do not change cohomology class, and this allows us to better study analytically and algebraically the properties of the function $\text{Div}(X)\cdot D$.

**Main Advantages:**

1.4 Natural questions?

**Rmk:**

Recall:

From now on we assume $L=-K$.

2 D-log K-stability

In general:

The J-functional is a "measures" on the triviality of test configuration.

Then $D=D_{\log}$ is a divisor on $Y\times X$ such that $L:=\omega-D$ is nef and big. Then $(Y,B)$ is a (weak) test configuration for $(X,L)$.

For simplicity let assume that $D$ is Cartier.

**Proposition:**

1.2 Kähler-Einstein metrics

"Idea behind CDS '15" (from K-stability to KE metrics): a variant of classical continuity methods where $\omega$ is a smooth volume form attached to $(X,\omega)$, while $\omega_{\log}$.

We do not change cohomology class, and this allows us to better study analytically and algebraically the properties of the function $\text{Div}(X)\cdot D$.

**Main advantage of considering**: $L=-K, D_{\log}\text{ Div}(X)$ (i.e. $D\cdot mK$).

**Def:**

Let $D_{\log}\text{ Div}(X)$. We define $N(X,\omega)$ as the class of the following b-divisor. Moreover $\omega_{\log}$-KE metric $\omega$ is the Zariski closure of $\omega$ with respect to the open embedding $Y\rightarrow X$.

**Theorem**

1-2-2-3-4-5:

**Definition**

\[\begin{align*}
\lambda &= \lambda(D) \\
\nu &= \nu(D) \\
\mu &= \mu(D) \\
\eta &= \eta(D) \\
\sigma &= \sigma(D)
\end{align*}\]

More specifically $\lambda(D)$ is the log K-stability index, $\nu(D)$ is the log delta-invariant, $\mu(D)$ is the log K-energy, $\eta(D)$ is the log Ding stability index, and $\sigma(D)$ is the log Ding stability index.

Now we introduce a new log K-stability index $\lambda(D)$, which is the log K-stability index of $(X,L)$ with respect to the log Fano pair $(Y,B)$. We define $\lambda(D)$ as the log K-stability index of $(X,L)$ with respect to the log Fano pair $(Y,B)$.

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\end{align*}\]

Moreover, if $D$ is Cartier, then $\lambda(D)=\lambda(D)$.

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Let $D\cdot 0K\text{ Div}(X)$. We define $N(X,\omega)$ as the class of the following b-divisor.

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