MATH1003

ASSIGNMENT 1
ANSWERS

1. Let $f(x) = \dfrac{x}{x+1}$. Then:

(i) $f(2+h) = \dfrac{2+h}{3+h}$

(ii) $f(x+h) = \dfrac{x+h}{x+h+1}$

(iii) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{1}{(x+1)(x+h+1)}$.

2. As a rational function, the function is defined whenever the denominator is non-zero. Since $x^2 + 3x + 2 = (x+2)(x+1)$, this is zero when $x = -2$ or -1. The domain is $\mathbb{R} \setminus \{-2, -1\}$.

3. (i) This is a rational function whose denominator is zero when $t = 2$. Hence it has domain $\mathbb{R} \setminus \{2\}$. By factorising the numerator, we see:

$$H(t) = \frac{(2-t)(2+t)}{2-t} = 2 + t.$$

Drawing the graph is now elementary.

(ii) $g(x)$ is defined whenever the denominator is non-zero. Hence the domain is $\mathbb{R} \setminus 0$. To sketch the graph, we consider the cases when $x < 0$ and $x > 0$ separately. From the definition of $|x|$ we see that:

$$g(x) = \begin{cases} -x/x^2 = -1/x, & \text{when } x < 0; \\ x/x^2 = 1/x, & \text{when } x > 0. \end{cases}$$

Drawing this graph should cause no problems.

(iii) By sketching the graph, we see that the domain is $\mathbb{R}$.

4. (i) $x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x-2)(x+2)(x^2+4)$. Hence:

$$\frac{x^4 - 16}{x - 2} = (x+2)(x^2+4).$$

Thus we see that the limit as $x \to 2$ is $4 \times 8 = 32$.

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(ii) When $x < 5$, so $x - 5$ is negative. As $x \to 5$ from the left, so $x - 5 \to 0$ from the left. Hence:

$$\lim_{x \to 5^-} \frac{6}{x-5} = -\infty.$$

(iii) For x sufficiently close to zero, $x - 1$ is negative, but $x^2(x+2)$ is positive. Hence:

$$\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty.$$