MATH1003

ASSIGNMENT 10
ANSWERS

1. Let x and d denote the lengths of the sides, in meters, of the rectangle. Then:

$$xd = 1000,$$
$$\Rightarrow \qquad d = \frac{1000}{x}.$$

The perimeter is given by $P(x) = 2x + 2d = 2(x + 1000/x)$. We wish to minimise $P(x)$.

$$\frac{dP}{dx} = 2 - \frac{2000}{x^2}.$$

The derivative is zero when $x = \pm 10\sqrt{10}$. Since x is a length, x must be positive. We shall investigate the behaviour of $P(x)$ as x tends to zero, and as x grows very large.

$$\lim_{x\to 0^+} 2\left(x + \frac{1000}{x}\right) = \infty$$
$$\lim_{x\to\infty} 2\left(x + \frac{1000}{x}\right) = \infty.$$

Hence the global minimum value of $P(x)$ occurs when $x = 10\sqrt{10}$. So the desired dimensions are $10\sqrt{10}\,\text{m} \times 10\sqrt{10}\,\text{m}$.

2. Let x and h denote the lengths, in meters, of the sides of the box, where the square base has sides of length $x\,$m. Then:

$$x \times x \times h = 12,$$
$$\Rightarrow \qquad h = \frac{12}{x^2}.$$

Let $C(x)$ denote the cost, in dollars, of the materials. Then:

$$\begin{aligned} C(x) &= 2 \times x^2 + 1 \times (x^2 + 4hx) \\ &= 3x^2 + \frac{48}{x} \\ &= 3\left(x^2 + \frac{16}{x}\right). \end{aligned}$$

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Since x is a length we require that $x > 0$. As x approaches zero, and as x grows very large, we see that:

$$\lim_{x\to 0^+} 3\left(x^2 + \frac{16}{x}\right) = \infty,$$
$$\lim_{x\to \infty} 3\left(x^2 + \frac{16}{x}\right) = \infty.$$

Differentiating $C(x)$ we obtain:

$$\frac{dC}{dx} = 3\left(2x - \frac{16}{x^2}\right).$$

This is zero when $2x^3 = 16$, i.e. when $x = 2$. Hence $C(x)$ must have a global minimum when $x = 2$. Thus in order to minimise the cost of materials, the box should have dimensions $2\,\mathrm{m} \times 2\,\mathrm{m} \times 3\,\mathrm{m}$.

3. Let x denote the number of members over 100. Then $0 \le x \le 160$. Let $R(x)$ denote the revenue. Then:

$$R(x) = (100 + x)(200 - x).$$

This is a quadratic with roots when $x = -100$ and when $x = 200$. Differentiating we obtain:

$$\begin{aligned}\frac{dR}{dx} &= 200 - x - 100 - x\\ &= 100 - 2x.\end{aligned}$$

Hence dR/dx is zero when $x = 50$. Since $R(50) = 150^2$ we know that this is the global maximum value of R. Hence the revenue is maximised when there are 150 members.

4. Let P denote the profit. Then:

$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= x^2 + 50x - 2x^3 - x^2 + 100x - 200\\ &= -2x^3 + 150x - 200.\end{aligned}$$

We have that $0 \le x \le 8$. At the end points,

$$\begin{aligned}P(0) &= -200,\\ P(8) &= -24.\end{aligned}$$

Differentiating $P(x)$ we obtain:

$$\frac{dP}{dx} = -6x^2 + 150.$$

This is zero when $x = \pm 5$. From the shape of the graph of $y = P(x)$ we see that there is a global maximum value when $x = 5$. Hence the manufacturer should produce five shovels per day.

5. Let $C(x)$ denote the total cost of producing x bicycles, where $x \geq 0$. Then:

$$C(x) = 1000 + 10x + \frac{25000}{x}.$$

Investigating the behaviour of $C(x)$ as x tends to zero and as x tends to infinity we see that:

$$\lim_{x \to 0^+} \left(1000 + 10x + \frac{25000}{x}\right) = \infty, \qquad \lim_{x \to \infty} \left(1000 + 10x + \frac{25000}{x}\right) = \infty.$$

Differentiating $C(x)$ gives:

$$\frac{dC}{dx} = 10 - \frac{25000}{x^2}.$$

This is zero when $x = \pm 50$. We thus see that $y = C(x)$ has a global minimum when $x = 50$, and so the manufacturer should produce fifty bicycles in order to minimise the total cost.