MATH1003
ASSIGNMENT 3
ANSWERS

1. (i) We can factorise the numerator:

$$x^2 - 81 = (x-9)(x+9) = (\sqrt{x}-3)(\sqrt{x}+3)(x+9).$$

Hence:

$$\frac{x^2-81}{\sqrt{x}-3} = (\sqrt{x}+3)(x+9).$$

Thus the limit is $(3+3)\times(9+9) = 108$.

(ii) Recall that:

$$|x| = \begin{cases} x, & \text{when } x \geq 0; \\ -x, & \text{when } x < 0. \end{cases}$$

Since $x \to -1$ we can insist that $x < 0$. Hence we wish to calculate:

$$\lim_{x\to -1} \frac{-x-1}{x+1} = \lim_{x\to -1} -\frac{x+1}{x+1} = -1.$$

2. In order for the function to be continuous, we need the two parts of the function to "meet" at the point $x = 4$. Thus we require:

$$4^2 - c^2 = 4(5+c).$$

Rearranging gives:

$$c^2 + 4c + 4 = 0.$$

Hence we see that $c = -2$.

3. The limit as $x \to \infty$ of $\cos x$ does not exist. For example, we have that $\cos 2k\pi = 1$ for all $k \in \mathbb{N}$, but $\cos 2(k+1)\pi = -1$ for all $k \in \mathbb{N}$.

4. Let $f(x) = (2+x)^3(1-x)(3-x)$. Since f is a polynomial, it is continuous for all $x \in \mathbb{R}$. Clearly $f(x) = 0$ when $x = -2$ (where we have a root with multiplicity three), and when $x = 1$ or 3. For large x, both $1-x$ and $3-x$ are negative, thus

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$f(x)$ is positive (+'ve $\times$−'ve $\times$−'ve = +'ve). Hence $\lim_{x\to\infty} f(x) = \infty$. For very negative x, both $1-x$ and $3-x$ are positive, and $(2+x)^3$ is negative. Hence:

$$\lim_{x\to-\infty} f(x) = -\infty.$$

Using this information we can draw an approximate sketch of $y = f(x)$.

5. Observe that, for all values of $x \neq 0$:

$$-1 \leq \cos\frac{\pi}{x} \leq 1.$$

Hence:

$$-\sqrt{x^5+3x} \leq \sqrt{x^5+3x}\,\cos\frac{\pi}{x} \leq \sqrt{x^5+3x}.$$

Now, clearly:

$$\lim_{x\to0} -\sqrt{x^5+3x} = 0 \qquad \text{and} \qquad \lim_{x\to0} \sqrt{x^5+3x} = 0.$$

Hence by the Squeeze Theorem we see that:

$$\lim_{x\to0} \left(\sqrt{x^5+3x}\,\cos\frac{\pi}{x}\right) = 0.$$

6. Notice that $x^2+2x-3 = (x+3)(x-1)$, so in order for the limit to exist, we need to cancel the term $x+3$ in the denominator with the numerator. Hence we require that $x+3$ divides $x^2+ax+a+3$. By long division, or otherwise, we obtain:

$$\begin{array}{r|l}
 & x + (a-3) \\ \hline
x+3 & x^2 + ax + a + 3 \\
 & x^2 + 3x \\ \hline
 & (a-3)x + (a+3) \\
 & (a-3)x + 3(a-3) \\ \hline
 & -2a + 12
\end{array}$$

Hence it must be that the remainder is zero. In other words, we require that:

$$-2a + 12 = 0.$$

Solving gives $a = 6$. When this is the case,

$$\begin{aligned}
\lim_{x\to-3} \frac{x^2+6x+9}{x^2+2x-3} &= \lim_{x\to-3} \frac{(x+3)(x+3)}{(x+3)(x-1)} \\
&= \lim_{x\to-3} \frac{x+3}{x-1} \\
&= 0.
\end{aligned}$$