

**MATH1003
ASSIGNMENT 3**

Suggested practice questions (the answers are in the back of the textbook):

- §2.3; 21, 27, 33, 37, 43, 55, 61.
- §2.5; 21, 35, 41, 45.

1. Calculate the following limits, if they exist:

(i) $\lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3}$,

(ii) $\lim_{x \rightarrow -1} \frac{|x| - 1}{x + 1}$.

2. Let $f(x)$ be the function given by:

$$f(x) = \begin{cases} x^2 - c^2, & \text{when } x < 4; \\ x(5 + c), & \text{when } x \geq 4. \end{cases}$$

For what values of the constant c is the function continuous?

3. Does the limit $\lim_{x \rightarrow \infty} \cos x$ exist? If not, why not?

4. Let $f(x) = (2 + x)^3(1 - x)(3 - x)$. Calculate:

$$\lim_{x \rightarrow -\infty} f(x) \quad \text{and} \quad \lim_{x \rightarrow \infty} f(x).$$

Sketch a graph of $y = f(x)$, making sure that you label the points of intersection with the axes.

5. By using the Squeeze Theorem, show that:

$$\lim_{x \rightarrow 0} \left(\sqrt{x^5 + 3x} \cos \frac{\pi}{x} \right) = 0.$$

6. Is there a constant a such that

$$\lim_{x \rightarrow -3} \frac{x^2 + ax + a + 3}{x^2 + 2x - 3}$$

exists? If so, find the value of a and the corresponding value of the limit.