

MATH1003
PRACTICE MID-TERM

This exam has five questions, with each question worth 5 marks.
The exam lasts for one-and-a-half hours. No calculators, books etc. are allowed.

1. Sketch the curve $y = |x^2 - 2|$. Find the equation of the tangent line to the curve at the point $(-1, 1)$. At what point does this tangent line cross the y -axis?

2. Using the rules of differentiation, differentiate the following functions:

(i) $y = (\tan x + x^2)^{3/2}$,

(ii) $u(t) = \frac{\sec t}{t}$,

(iii) $f(\theta) = \sqrt{\theta}e^{2\theta} + 1$,

(iv) $y = \sin^{-1}(2x)$,

(v) $g(t) = \frac{\cos 2t}{t^2}$.

3. The piece-wise function $s(x)$ is given by:

$$s(x) = \begin{cases} \frac{1}{x}, & \text{when } x < -1; \\ x^2, & \text{when } -1 \leq x < 2; \\ 2x, & \text{when } x \geq 2. \end{cases}$$

(i) Sketch a graph of $y = s(x)$. State the domain and range.

(ii) Evaluate the following limits:

(a) $\lim_{x \rightarrow -1^-} s(x)$,

(b) $\lim_{x \rightarrow -1^+} s(x)$,

(c) $\lim_{x \rightarrow 2^-} s(x)$,

(d) $\lim_{x \rightarrow 2^+} s(x)$.

(iii) When is $s(x)$ continuous? Give a brief justification for your answer.

4. (i) State a definition of derivative in terms of limits.
(ii) Using your definition in (i), calculate the derivative of:

$$f(t) = 9 + 5t - 2t^2.$$

- (iii) When is $f'(t) = 0$?

5. Evaluate the following limits, or give a reason why they do not exist:

(i) $\lim_{t \rightarrow 3^+} \frac{2}{\sqrt{t-3}}$,

(ii) $\lim_{x \rightarrow 0} \left(7x + \frac{\sin(3x)}{7x} \right)$,

(iii) $\lim_{t \rightarrow 2} \frac{t^2 - 3t + 2}{t^2 - t - 2}$,

(iv) $\lim_{x \rightarrow \infty} \frac{x^2 - 5}{3x^2 + 2x - 1}$.