MATH1003
QUIZ 6 – MOCK FINAL

This quiz has ten questions, with each question worth 5 marks.
The quiz lasts for three hours. No calculator, textbooks, or other notes are allowed.

1. Find the derivative of the following functions:

(i) $f(x) = 5x^3 + \dfrac{7}{x^2} + 2\sqrt{x}$,

(ii) $g(x) = (x^2 + 3)\sin x$,

(iii) $y = \cos x^3 + 5$,

(iv) $f(x) = (x^3 + 3)^4(3x^2 + 7)^5$,

(v) $f(t) = \dfrac{\sin 3t}{\cos 2t}$,

(vi) $y = \sin^{-1}\sqrt{x} + \cosh x$,

(vii) $y = \ln(\ln(2 + x)) + e^{3x^2}$.

2. The function

$$f(x) = \begin{cases} x, & \text{when } 0 \le x < 1; \\ 0, & \text{when } x = 1 \end{cases}$$

is zero at $x = 0$ and $x = 1$ and differentiable on $(0, 1)$, but its derivative on $(0, 1)$ is never zero. How can this be? Doesn't Rolle's Theorem say that the derivative must be zero somewhere in $(0, 1)$? Give reasons for your answer.

3. (i) If a function $g(x)$ is continuous on $[a, b]$, differentiable on (a, b), and such that $g(a) = g(b)$, what does Rolle's Theorem tell us?

(ii) For the function

$$f(x) = x^3 + 2x - 2,$$

by applying the Intermediate Value Theorem, show that f has a root in the interval $0 < x < 1$.

(iii) Show that $f'(x)$ is always positive.

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(iv) Let a in the interval $(0,1)$ be such that $f(a)=0$. Suppose that there exists b such that $a \neq b$ and $f(b)=0$. By applying Rolle's Theorem and considering your answer in (iii), explain why no such b can exist.

4. (i) Prove that $(\cosh x)^2 - (\sinh x)^2 = 1$.

(ii) Let $y = \sinh^{-1} x$ be the inverse of $\sinh x$. Show that:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}.$$

(Hint: You may wish to consider $\sinh y = x$.)

(iii) When is the tangent to $y = \sinh^{-1} x$ parallel with the line $y = x$?

5. Find an equation of the tangent line to the curve

$$y = \frac{1}{\sin x + \cos x}$$

at the point $(0,1)$.

6. Use implicit differentiation to find $\frac{dy}{dx}$ given that:

$$ye^{x^2} = xe^{y^2}.$$

7. Evaluate the following limits. Use l'Hospital's Rule if appropriate. If the rule does not apply, explain why and evaluate the limit by other means.

(i) $\lim_{x\to\infty} \frac{e^x}{x}$,

(ii) $\lim_{x\to 0} \frac{x+\sin x}{x+\cos x}$,

(iii) $\lim_{x\to 0^+} \sin x \ln x$.

(Hint: You may find it useful to recall that $\frac{d}{dx}\csc x = -\csc x \cot x$.)

8. (i) Find the horizontal and vertical asymptotes of the curve

$$y = \frac{2x^2+1}{x^2-x}.$$

(ii) Using your results from (i), draw a rough sketch of this curve.

9. For the function $y = x^3 - \frac{3}{2}x^2 - 6x$, find the following:

(i) The domain.

(ii) The range.

(iii) The intervals of increase.

(iv) The intervals of decrease.

(v) The intervals of concave up.

(vi) The intervals of concave down.

(vii) Sketch the function.

10. An open rectangular box of volume 400 cubic centimetres has a square base and a partition down the middle (see Figure 1). Find the dimensions of the box for which the amount of material needed to construct the box is as small as possible.

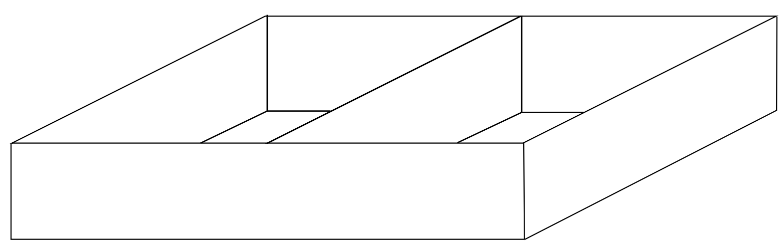

FIGURE 1. An open rectangular box with dividing partition.