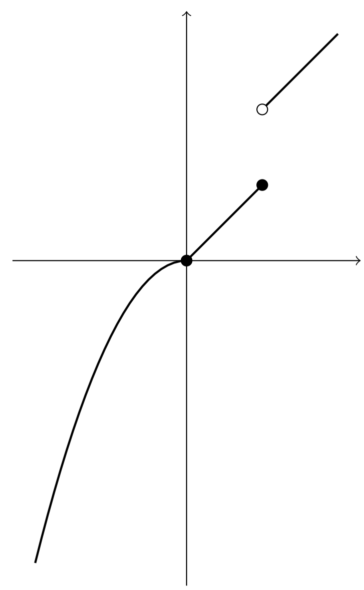
1. Let $f(x) = \begin{cases} -x^2 & x < 0 \\ x & 0 \leq x \leq 1 \\ x+1 & x > 1 \end{cases}$

   (a) Sketch the graph of f. What is the range of f.

   You should fill lable the axes and fill put tics on them to indicate the scale. The range of f is all real numbers except those in the interval $(1, 2]$

   (b) For which values of a is f discontinuous at $x = a$? Give a reason why f is not continuous at this value(s).

   f is continuous at all values except $x = 2$. At this point, the limit does not exist. Specifically, $\lim_{x \to a^-} f(x) = 1$ and $\lim_{x \to a^+} f(x) = 2$ are not equal.

   (c) See (b).

2. Find the derivatives of the following functions.

   (a) $y = (1+x^2)^{100}$, $y' = 100(1+x^2)^{99}(2x) = 200x(1+x^2)^{99}$.

   (b) $y = (x+1)\sin(x)$,

$$\begin{aligned} y' &= (1)\sin(x) + (x+1)(-\cos(x)) && \text{product rule} \\ &= \sin(x) - (x+1)\cos(x) \end{aligned}$$

   (c) $y = \dfrac{\sqrt{1-x^2}}{x}$, First, let $f(x) = \sqrt{1-x^2}$ and $g(x) = x$. Using the chain rule,

$$f'(x) = -\frac{x}{\sqrt{1-x^2}},$$

   and clearly $g'(x) = 1$. Using the quotient rule and the above derivatives

$$\begin{aligned} y' &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \\ &= \frac{-\dfrac{x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}}{x^2} \\ &= -\frac{1}{x^2\sqrt{1-x^2}} \end{aligned}$$

   (d) $y = (x^2+1)\sqrt[3]{x^2+2}$, First, let $f(x) = x^2+1$ and let $g(x) = \sqrt[3]{x^2+2}$. Then $f'(x) = 2x$, and, using the chain rule, $g'(x) = \dfrac{2x}{3(x^2+2)^{\frac{2}{3}}}$. Hence, using the product rule,

$$\begin{aligned} y' &= f'(x)g(x) + f(x)g'(x) \\ &= 2x\sqrt[3]{x^2+2} + \frac{2x(x^2+1)}{3(x^2+2)^{2/3}} \\ &= \frac{6x(x^2+2) + 2x(x^2+1)}{3(x^2+2)^{2/3}} \\ &= \frac{8x^3+14x}{3(x^2+2)^{2/3}} \end{aligned}$$

3. Evaluate the following limits:

   (a) $\displaystyle\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} = \lim_{x\to 25} \frac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)} = \frac{1}{10}$.

   (b) $\displaystyle\lim_{x\to\infty} \frac{3x^3-2x^2}{x^3+3x-1} = 3$.

   (c) $\displaystyle\lim_{x\to 2^+} \frac{x^2-4}{x^2-4x+4} = \lim_{x\to 2^+} \frac{(x-2)(x+2)}{(x-2)^2} = \lim_{x\to 2^+} \frac{(x+2)}{(x-2)} = +\infty$

4. Let $f(x) = \sqrt{2x+1}$ and $g(x) = x^2$.

   (a) The domain of f is all real numbers x such that $x \geq -1/2$.

   (b) $f(g(x)) = \sqrt{2x^2+1}$

   (c) The domain of $f(g(x))$ is all real numbers.

5. Let $f(x) = x^3 - 3x^2 - 8x$.

   (a) Find the equation of the tangent line to the curve $y = f(x)$ at the point $(1, -10)$. This is the line with slope $m = f'(1) = 3(1)^2 - 6(1) - 8 = -11$ passing through the point $(1, -10)$. An equation for this line is

$$y + 10 = -11(x - 1)$$

   (b) The tangent line has slope one whenever $f'(x) = 1$. Since $f'(x) = 3x^2 - 6x - 8$, this occurs when $x = 3$ or $x = -1$.

6. Compute $f'(x)$ from the definition of the derivative when $f(x) = 3x - x^2$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\big(3(x+h) - (x+h)^2\big) - \big(3x - x^2\big)}{h} \\
&= \lim_{h\to 0} \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h} \\
&= \lim_{h\to 0} \frac{(3-2x)h - h^2}{h} \\
&= \lim_{h\to 0} ((3-2x) - h) \\
&= 3 - 2x
\end{aligned}$$